ACCURATE DISTORTION MEASUREMENT FOR B-SPLINE-BASED SHAPE CODING

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ABSTRACT

In this paper, we present a new contour point distortion measurement, called accurate distortion measurement for B-spline-based shape coding (ADMBSC). Different from existing distortion measurements containing approximation, quantization or parameterization, our distortion is defined as the shortest distance from the original B-spline to the associated contour point. This is in line with the subjective-based objective quality metric. Geometric relationships are introduced to simplify computation, followed by a hybrid admissible distortion checking algorithm to reduce execution time. Theoretical analysis and experimental results demonstrate that when the operational rate-distortion optimal shape coding framework under the minimum-maximum criterion is applied, the ADMBSC can lead to the smallest bit-rate among all the distortion measurements that can guarantee the admissible distortion. Moreover, if the original contour has N_C points, it takes only $O(N_C)$ time for segment distortion measuring paradigms, whose computational complexity is the same as the lowest one among the existing distortion measurements.

Index Terms—Video coding, image coding, image quality

1. INTRODUCTION

Shape coding is the fundamental technology to implement the object-oriented video applications, such as storage, retrieval, editing and interaction [1]. Among existing shape coding techniques, B-spline-based shape coding can reflect the local contour characteristics with nature reconstruction appearance, so it has unique advantages in the object-oriented video applications [2].

Distortion measurement is a key component in B-spline-based shape coding. It closely relates to the actual reconstruction quality and dominates the overall execution time, so it plays a very important role in both rate-distortion performance and computational efficiency [3].

Existing distortion measurements can be computed from three models, whose problems are shown as follows:

Polyline model: The *shortest absolute distance* (SAD) [1] and *accurate distortion measurement for shape coding* (ADMSC) [3] [4] belong to this model. It inappropriately measures the distortion on the polyline approximated B-splines. Therefore, it fails to correctly measure the actual distortion as shown in Fig. 1(a). Moreover, all the polylines should be calculated for each associated contour point. Thus, the overall computational complexity is $O(N_C^2)$, where N_C denotes the number of the original contour points.

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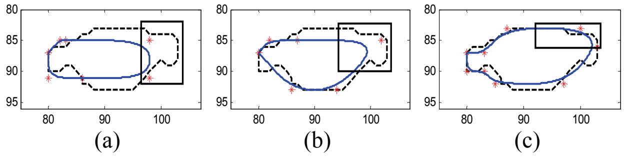


Fig. 1. Results for the *Lip* region of the 31st frame of the *MissAmerica.qcif* sequence with the admissible distortion pair $T_{\max} = T_{\min} = 2pels$ (a) Polyline model with $CP = 7$, $R = 48bits$, $D = 5.60pels$, $T = 2.51secs$, (b) Band model with $CP = 6$, $R = 44bits$, $D = 3.91pels$, $T = 1.99secs$. (c) Parameterization model with $CP = 9$, $R = 60bits$, $D = 1.72pels$, $T = 0.98secs$. (**Notation**—*CP*: number of control point; *R*: bit-rate; *D*: peak distortion; *T*: execution time **Legend**—*solid line*: approximating B-splines; *dashed line*: original contour; *asterisk*: control point)

Band model: The *distortion band* (DB) [1] and *tolerance band* (TB) [5] belong to this model. It checks whether the quantized B-spline lies inside the band. As a result, it ignores the sharp futures as shown in Fig. 1(b). Since the band creation can be computed independently, the overall time complexity is $O(N_C)$.

Parameterization model: The *distortion measurement using chord-length parameterization* (DMCLP) [6] and *distortion measurement using arc-length parameterization* (DMALP) [7] belong to this model. It calculates the distortion as the distance between the associated contour point and the corresponding B-spline curve point. Although the admissible distortion is upheld, it results in extra control points and thereby leading to additional encoding bits as illustrated in Fig. 1(c). On computational aspect, since both of the parameterized B-spline point and the distance need to be calculated only once for each associated contour point, the overall time complexity is also $O(N_C)$.

The above problems motivate us to develop a novel distortion measurement, called *accurate distortion measurement for B-spline-based shape coding* (ADMBSC), which should satisfy the following three requirements: 1) it can guarantee the admissible distortion, 2) the encoding bits should be as few as possible, and 3) the overall execution time should be as short as possible.

To achieve these aims, we construct a model that can accurately measure the actual distortion. It was reported that the actual distortion for subjective-based objective reconstruction quality assessment is the minimal Euclidean distance between each contour point and the reconstruction contour [1]. Therefore, we model the distortion as the shortest distance from the approximating B-spline to the associated contour point. Then this model is in accordance with the actual distortion.

To simplify the computation, we introduce some geometric relationships. A key observation is that the shortest path in the above model is perpendicular to the tangent vector of the approximating B-spline. Consequently, we construct a parametric equation by setting their dot product to zero. To accelerate this implementation, we present a hybrid admissible distortion checking algorithm.

The rest of this paper is organized as follows. Sect. 2 describes our proposed ADMBSC. Sect. 3 investigates the geometric relationships and presents a method to compute ADMBSC. Sect. 4 shows the advantages of ADMBSC on both rate and computational aspects. Full experimental analysis is provided in Sect. 5. Finally, some conclusion remarks and future work are given in Sect. 6.

2. ACCURATE DISTORTION MEASUREMENT MODEL

From the above analysis, we find that the polyline model and the band model do not define the distortion on the original approximating B-spline. Therefore, they are far away from satisfying the reconstruction quality assessment [1]. In contrast, the parameterization model defines the distortion on the approximating B-spline and is close to satisfying all the performance requirements stated in Sect. 1, except the bit-rate requirement. So it is feasible to construct a required model using the reference of this model. The challenge is how to make full use of its advantages and avoid its disadvantages. We find that this extra bits problem is due to the inappropriate curve points produced by parameterization techniques. Thus, the calculated distortion may be larger than its actual value, which leads to discarding of the candidate approximating B-splines whose actual distortion upholds the admissible distortion with fewer bits as shown in Fig. 2(a). If we can generate a B-spline point $\mathbf{q}^*$, the distance from which to the associated contour point $\mathbf{c}$ is the shortest distance from the approximating B-spline Q_{BS} to $\mathbf{c}$, the inherent shortcoming of parameterization model can be compensated as this definition is consistent with the quality assessment metric.

***Definition 1**:* Let $\mathbf{p}_0$, $\mathbf{p}_1$ and $\mathbf{p}_2$ be three consecutive control points, then the ADMBSC from the approximating B-spline Q_{BS} to the associated contour point $\mathbf{c}$ can be formulated as

$$ADMBSC = \min_{\mathbf{q}\in Q_{BS}} \|\mathbf{cq}\|_2 = \min_{t\in[0,1]} \|\mathbf{cq}(t)\|_2, \tag{1}$$

where $\|\cdot\|_2$ denotes the L_2-norm,

$$\mathbf{q}(t) = \begin{bmatrix} t^2 & t & 1 \end{bmatrix} \cdot \begin{bmatrix} 0.5 & -1 & 0.5 \\ -1 & 1 & 0 \\ 0.5 & 0.5 & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \end{bmatrix}, t\in[0,1] \tag{2}$$

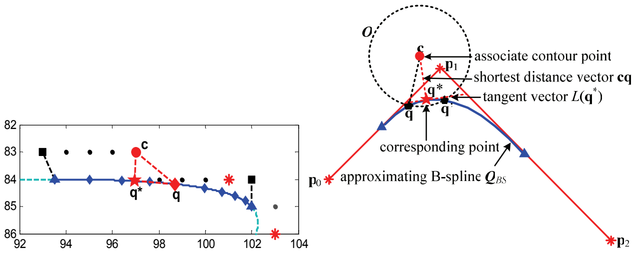
is a standard quadratic basis uniform non-rational B-spline form [2], $t^* = \arg\min_{t\in[0,1]} \|\mathbf{cq}(t)\|_2$ is called corresponding parameter, $\mathbf{q}^* = \mathbf{q}(t^*)$ is called corresponding point, $\mathbf{cq}^*$ is called shortest distance vector as shown in Fig. 2(b).

3. COMPUTATIONAL METHOD

A straightforward method to compute ADMBSC is to find the explicit expression of $\|\mathbf{cq}\|_2$ firstly, and then obtain its minimum value through mathematical analysis. However, this method is quite complicated. Thus, we consider it on geometric aspect.

Note that these geometric relationships depend on three relative positions between $\mathbf{c}$ and Q_{BS}: 1) $\mathbf{c}\in Q_{BS}$, so $\mathbf{q}^* = \mathbf{c}$, 2) $\mathbf{c}\notin Q_{BS}$ and $\mathbf{q}^*$ is on the open Q_{BS}; and 3) $\mathbf{c}\notin Q_{BS}$ and $\mathbf{q}^*$ is at the end of Q_{BS}. Among them, 2) is the most general case, so we start with 2) and expect that 1) and 3) can be integrated into the formula derived from 2). The theorem below follows this idea.

Theorem 1: When $\mathbf{c}\notin Q_{BS}$ and $\mathbf{q}^*$ is on the open Q_{BS}, $\mathbf{cq}^* \perp L(\mathbf{q}^*)$, where $L(\cdot)$ denotes the tangent vector of Q_{BS} and $\perp$ denotes the perpendicularity as shown in Fig. 2(b).



(a) Parameterization model (b) Accurate model

Fig. 2. (a) Alternative approximating B-spline for the contour segment highlighted in Fig. 1(c) with fewer bits. Although its actual distortion upholds the admissible distortion, its parameterization model based distortion is $\|\mathbf{cq}\|_2 = 2.07$*pels*, which exceeds the admissible distortion. Thus, this approximating B-spline is unpermitted and more bits are required. If we define $\|\mathbf{cq}^*\|_2$ as distortion instead, this situation can be avoided. (b) Definition of our accurate model. (**Legend** — *solid line*: approximating B-spline; *asterisk*: control point; *round*: associated contour point; *diamond*: parameterized curve point; diamond: *pentagram*: corresponding point)

Proof: Draw a circle $\boldsymbol{O}$ centered at $\mathbf{c}$ and passing through $\mathbf{q}^*$ as shown in Fig. 2(b). If we can prove that $\boldsymbol{O}$ is tangent to Q_{BS} at $\mathbf{q}^*$, then $L(\mathbf{q}^*)$ is also tangent to $\boldsymbol{O}$ at $\mathbf{q}^*$, which yields $\mathbf{cq}^* \perp L(\mathbf{q}^*)$.

Assume, by contradiction, that $\boldsymbol{O}$ is not tangent to Q_{BS}. Then, $\boldsymbol{O}$ must intersect Q_{BS} at $\mathbf{q}^*$ (referred to as $\mathbf{q}$ for distinction from the actual $\mathbf{q}^*$). Thus, there is one half neighborhood of $\mathbf{q}$ on Q_{BS} inside $\boldsymbol{O}$ (right neighborhood ($\mathbf{q}$, $\mathbf{q}$') in Fig. 2(b) as an example), with the distance from the curve point belonging to ($\mathbf{q}$, $\mathbf{q}$') to $\mathbf{c}$ less than $\|\mathbf{cq}\|_2$. It conflicts with the definition of ADMBSC that $\|\mathbf{cq}\|_2$ is the shortest distance from Q_{BS} to $\mathbf{c}$. Hence, Theorem 1 is proved. □

Based on Theorem 1, in the first two geometric relationship cases, $\mathbf{q}^*$ should satisfy the following equation:

$$\mathbf{cq}\cdot L(\mathbf{q}) = 0, \tag{3}$$

where $\cdot$ denotes the dot product, and in the third case, the boundary conditions should be added to the candidate parameter solution space of Eq. (3) to find the final ADMBSC.

Following this computational framework, we specify the general procedures as follows: 1) find the expressions of $\mathbf{cq}$ and $L(\mathbf{q})$, where $\mathbf{q}\in Q_{BS}$, 2) set the dot product of $\mathbf{cq}$ and $L(\mathbf{q})$ to zero, 3) resolve the obtained parametric equation and find the candidate parameter solution space Γ, and 4) add the boundary condition set $\{0, 1\}$ to Γ and find the minimum value of $\|\mathbf{cq}(t)\|_2$ in Γ as the solution to Eq. (1). The procedures in detail are shown as follows.

1) Rewrite Eq. (2) as a function w.r.t. t, such that

$$\mathbf{cq} = \mathbf{q}(t) - \mathbf{c} = (\mathbf{p}_0/2 + \mathbf{p}_2/2 - \mathbf{p}_1)t^2 + (\mathbf{p}_1 - \mathbf{p}_0)t + \mathbf{p}_0/2 + \mathbf{p}_1/2 - \mathbf{c}$$
$$= \mathbf{A}t^2 + \mathbf{B}t + \mathbf{C}\,, \tag{4}$$

where

$$\mathbf{A} = \mathbf{p}_0/2 + \mathbf{p}_2/2 - \mathbf{p}_1,\ \mathbf{B} = \mathbf{p}_1 - \mathbf{p}_0,\ \text{and}\ \mathbf{C} = \mathbf{p}_0/2 + \mathbf{p}_1/2 - \mathbf{c}, \tag{5}$$

and derive $L(\mathbf{q})$ by differentiating Eq. (2) w.r.t. t

$$L(\mathbf{q}) = \mathbf{q}'(t) = 2(\mathbf{p}_0/2 + \mathbf{p}_2/2 - \mathbf{p}_1)t + (\mathbf{p}_1 - \mathbf{p}_0) = 2\mathbf{A}t + \mathbf{B}. \tag{6}$$

2) Substitute Eq. (4) and Eq. (6) in Eq. (3)

$$\begin{aligned}\mathbf{cq}\cdot L(\mathbf{q}) &= 2\mathbf{AA}^T t^3 + 3\mathbf{AB}^T t^2 + (2\mathbf{AC}^T + \mathbf{BB}^T)t + \mathbf{BC}^T \\ &= at^3 + bt^2 + ct + d = 0, \qquad t\in(0,1)\end{aligned} \tag{7}$$

where ^{T} denotes the transpose and

$$a = 2\mathbf{AA}^T,\ b = 3\mathbf{AB}^T,\ c = 2\mathbf{AC}^T + \mathbf{BB}^T,\ \text{and}\ d = \mathbf{BC}^T. \tag{8}$$

3) Find Γ that contains all the solutions to Eq. (7). Here we need to discuss two cases.

a) When $a = 0$, $\mathbf{AA}^T = 0$, which yields $\mathbf{A} = (0, 0)$ and $b = 3\mathbf{AB}^T = 0$. According to Eq. (8), it is impossible that $\mathbf{B} = (0, 0)$; otherwise $\mathbf{p}_0 = \mathbf{p}_1 = \mathbf{p}_2$, which is usually forbidden by the shape coding algorithms. Therefore, $c = \mathbf{BB}^T > 0$. Then Eq. (7) can be rewritten as $ct + d = 0$, $c>0$, which yields $t = -d/c$. Next, we check whether $t\in(0, 1)$ to obtain Γ.

b) When $a \neq 0$, then Eq. (7) is a standard form of cubic equation, so we can apply the Cartan's formula [8] to resolve it. Similarly, we check whether $t\in(0, 1)$ to obtain Γ.

4) Add {0, 1} to Γ and calculate the distortion as follows:

$$ADMBSC = \min_{t\in\Gamma\cup\{0,1\}} \left\|\mathbf{cq}(t)\right\|_2 . \quad (9)$$

When ADMBSC of the associated contour points are computed, the segment distortion between Q_{BS} and original contour can be obtained by either a peak or mean-squared (MS) distortion measuring paradigm [1] with $O(N_C)$ time. The overall distortion can be given by the minimum-maximum (MINMAX) criterion in line with subjective assessment [9].

4. ADVANTAGES OF ADMBSC

As stated in Sect. 2, our ADMBSC can address the extra bits problem existed in parameterization model. Here we theoretically justify this advantage.

Theorem 2: When the operational-rate-distortion (ORD) optimal shape coding framework under the MINMAX criterion [2] is used, ADMBSC requires the smallest bit-rate among all the distortion measurements that can guarantee the admissible distortion.

Proof: Let T denote the admissible distortion for the contour points and DM denote any distortion measurement that can guarantee the given T. Let D_T(DM) denote the actual distortion given by DM under T and $R_{\mathrm{DM}}(T)$ denote the corresponding rate, when the ORD optimal framework under the MINMAX criterion is applied. If we can prove $R_{\mathrm{ADMBSC}}(T) \leq R_{\mathrm{DM}}(T)$, then Theorem 2 is verified.

Note that DM can guarantee T, then $D_T(\mathrm{DM}) \leq T$. Note that ADMBSC is in line with the actual distortion, so $R_{\mathrm{ADMBSC}}(D_T(\mathrm{DM}))$ is the minimum rate under D_T(DM), which is no greater than the rate $R_{\mathrm{DM}}(T)$ that is also under D_T(DM). It was proven in [10] that the rate $R_{\mathrm{ADMBSC}}(T)$ is a nonincreasing function of T. Thus, $D_T(\mathrm{DM}) \leq T$ implies that $R_{\mathrm{ADMBSC}}(T) \leq R_{\mathrm{ADMBSC}}(D_T(\mathrm{DM})) \leq R_{\mathrm{DM}}(T)$. □

As stated in Sect. 3, the computational complexity of ADMBSC is the same as that of DMCLP. However, for each associated contour point, a cubic equation needs to be solved in the implementation of ADMBSC. Therefore, ADMBSC consumes much more execution time than DMCLP. Nevertheless, in the ORD optimal framework under the MINMAX criterion, it is only necessary to check whether the segment distortion upholds the admissible distortion. Recalling their definitions, we find that the value of DMCLP is always no less than that of ADMBSC. That is, if DMCLP upholds the admissible distortion, ADMBSC will also uphold it; otherwise, ADMBSC should be calculated for further checking. Following this idea, we develop a *hybrid admissible distortion checking algorithm* (Hybrid ADMBSC) by adding an additional ADMBSC judgment to the original DMCLP checking algorithm. Consequently, a considerable part of execution time consumed by *pure ADMBSC checking algorithm* (Pure ADMBSC) can be saved as it is unnecessary to invoke ADMBSC for all the associated contour points. The above advantages on both rate and execution time aspects will be further demonstrated in Sect. 5.

5. EXPERIMENTAL RESULTS

This section conducts experiments to answer the question: to what extent does the performance of ADMBSC fulfill the requirements stated in Sect. 1? The *Neck* region of the 31st frame of the *MissAmerica.qcif* sequence is used as our input and the ORD optimal shape coding framework under the MINMAX criterion is applied as our platform.

The first set of experiments focuses on the subjective reconstruction quality. The various results with the admissible distortion setting $T_{\max} = T_{\min} = 2$*pels* are displayed in Figs. 3(a)-(f). Figures 3(a) and (c) show that both SAD and DB/TB ignore the relatively sharp portions, which play a very important role in reconstruction quality. It demonstrates that SAD and DB/TB cannot guarantee the admissible distortion and produce unacceptable quality impairments. By contrast, the other four distortion measurements, namely ADMSC, DMCLP, DMALP and our proposed ADMBSC, can reconstruct similar contours with tolerable perceptual distortions. They mandate 9, 12, 10 and 9 control points, which indicates that ADMBSC probably results in the smallest bit rate.

The next set of experiments concentrates on the objective peak distortion and execution time. The numerical results with different admissible distortion pairings are summarized in Table 1. For simplicity, we focus on the 5th row with $T_{\max} = T_{\min} = 2$*pels* as an example. SAD and DB/TB only need 65 and 66*bits*, but generate peak distortions of 6.52 and 5.24*pels*, despite the distortion bound is 2*pels*. While by comparison, ADMSC, DMCLP, DMALP and our proposed ADMBSC all uphold the admissible distortion at the cost of 69, 83, 75 and 69*bits*. It validates that our proposal can satisfy the first two requirements presented in Sect. 1. On execution time aspect, SAD and ADMSC require 5.01 and 10.17*secs*, while DB/TB, DMCLP, DMALP and Hybrid ADMBSC need 3.78, 2.25, 3.81, 3.38*secs*. It is consistent with the analysis in Sect. 1 that the former ones incur $O(N_C^2)$ time while the latter ones incur only $O(N_C)$ time. Although DB/TB mandates the shortest execution time, it cannot guarantee the admissible distortion as described above. Despite that Pure ADMBSC also takes $O(N_C)$ time, it demands 8.03*secs*, which is longer than SAD. It agrees with the analysis in Sect. 4 that the computation of ADMBSC for each associated contour point is very time consuming. On the other hand, Hybrid ADMBSC only consumes 3.38*secs*. It saves 4.65*secs*, which is more than one half of that consumed by Pure ADMBSC. Thus, it reveals the necessity of developing Hybrid ADMBSC. Moreover, the execution time of Hybrid ADMBSC is greater than that of DMCLP but less than that of DMALP, except in the case when $T_{\max} = T_{\min} = 1$*pel*. Furthermore, basically, the larger the admissible distortion is, the closer the execution time of Hybrid ADMBSC is to that of DMCLP. It further substantiates the priority of Hybrid ADBMSC on execution time aspect.

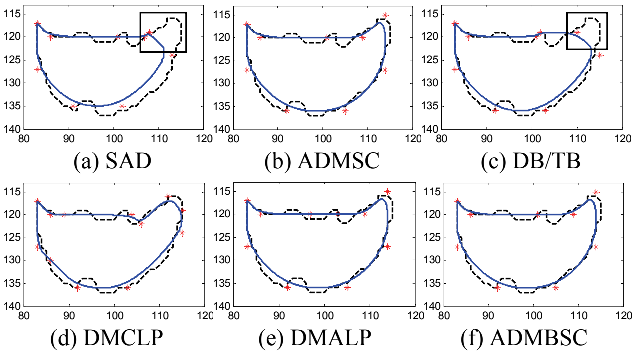
To conclude the experimental analysis, Table 2 presents a comparative summary of the key performance features and characteristics of the different distortion measurements. It verifies that ADMBSC is accurate and always guarantee the admissible distortion with the smallest bits, while the other measurements cannot satisfy at least two or more quality criteria. From a computational speed standpoint, ADMBSC mandates $O(N_C)$ time, which is the same as the lowest time complexity among all the examined distortion measurements. In addition, it can also successfully operate in both a peak and MS distortion measuring paradigms. Thus, we can draw a conclusion that our proposal can fulfill all the requirements proposed in Sect. 1.

Table 1 Numerical results for the *Neck* region (with obtained distortion whose values exceed the admissible peak distortion are highlighted) with different admissible distortion pairs ($T_{\max}$ & $T_{\min}$ in *pels*) using various B-spline-based segment distortion measurements.

Admissible distortion	Bit rate (*bits*), Peak distortion (*pels*), Execution time (*secs*)					
	SAD	***ADMSC***	***DB/TB***	***DMCLP***	***DMALP***	***ADMBSC (Pure/Hybrid)***
$T_{\max}$=1, $T_{\min}$=1	72, **4.37**, 4.73	118, 1.00, 7.95	79, **2.07**, 3.66	147, 1.00, 2.08	125, 1.00, 3.33	118, 1.00, 5.95 / 4.23
$T_{\max}$=2, $T_{\min}$=1	65, **6.52**, 4.85	78, 1.76, 9.45	66, **5.24**, 3.79	89, 1.69, 2.26	83, 1.72, 3.75	78, 1.76, 7.59 / 3.25
$T_{\max}$=2, $T_{\min}$=2	65, **6.52**, 5.01	69, 2.00, 10.17	66, **5.24**, 3.78	83, 1.80, 2.25	75, 2.00, 3.81	69, 2.00, 8.03 / 3.38
$T_{\max}$=3, $T_{\min}$=1	65, **6.52**, 5.04	69, 2.07, 10.98	61, **6.68**, 3.87	69, 2.00, 2.38	69, 2.07, 4.03	69, 2.07, 8.75 / 2.87
$T_{\max}$=3, $T_{\min}$=2	65, **6.52**, 5.05	69, 2.07, 11.10	61, **6.68**, 3.87	69, 2.07, 2.38	69, 2.07, 4.08	69, 2.07, 8.90 / 2.83

Table 2 Comparative chart for the various B-spline-based segment distortion measurements.

Quality criteria	*SAD*	*ADMSC*	*DB/TB*	*DMCLP*	*DMALP*	*ADMBSC*
Always reflects the actual distortion	NO	NO	NO	NO	NO	YES
Guaranteed admissible distortion	NO	NO	NO	YES	YES	YES
Guaranteed admissible distortion with the smallest bit-rate	NO	NO	NO	NO	NO	YES
Computational complexity	$O(N_C^2)$	$O(N_C^2)$	$O(N_C)$	$O(N_C)$	$O(N_C)$	$O(N_C)$
Distortion type: peak or MS	Both	Both	Peak	Both	Both	Both



(a) SAD (b) ADMSC (c) DB/TB

(d) DMCLP (e) DMALP (f) ADMBSC

Fig. 3. Reconstruction results for the *Neck* region with $T_{\max}=T_{\min}=2$*pels*.

6. CONCLUSION AND FUTHER WORK

This paper proposes a new distortion measurement, called ADMBSC, which can be used in any B-spline-based shape coding framework. It has the following four characteristics: 1) it is in line with the subjective-based objective peak distortion assessment, and thus it is consistent and reliable, 2) it totally avoids the approximation, quantization or parameterization process on the original B-splines, so it is accurate and effective; 3) when the B-spline-based ORD optimal shape coding framework under the MINMAX criterion is exploited, it can guarantee the admissible distortion with the smallest bit-rate, and 4) it takes only $O(N_C)$ time, which is the same as the lowest computational complexity among all the existing distortion measurements. Consequently, it is competitive among all the existing distortion measurements.

Besides the quadratic B-spline, the higher order B-splines can also be used for the vertex-based shape coding. As a result, we can extend our work into the B-splines with higher orders. In fact, the main idea of our model and computational method can be used as the reference model and computational method, since they share the same intrinsic characteristics. The main difference among them is that the order of derived parameter equations will be higher. Thereby, their solution derivation will be more complicated.

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