Accurate Distortion Measurement Using Analytical Model for the B-Spline-Based Shape Coding*

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Existing distortion measurements for the B-spline-based shape coding include approximation, quantization, or parameterization process, so they are approximate techniques. They may inaccurately predict the actual distortion value, which motivates us to construct a model that can accurately measure the actual distortion. It was reported that the actual distortion for reconstruction quality assessment is the minimal Euclidean distance between each associated contour point and the reconstruction contour. Therefore, we generally define our *accurate distortion measurement using analytical model* (ADMAM) of the *associated contour point* $\mathbf{c}$ from the *approximating B-spline* $\boldsymbol{Q}_{BS}$ as

$$ADMAM = \min_{\mathbf{q}\in\boldsymbol{Q}_{BS}} \left\|\mathbf{cq}\right\|_2, \tag{1}$$

where $\mathbf{q}^* = \arg\min_{\mathbf{q}\in\boldsymbol{Q}_{BS}} \left\|\mathbf{cq}\right\|_2$ is called the *corresponding point* and $\left\|\cdot\right\|_2$ is the L_2-norm.

For simplification, we compute this model from planar geometric aspect. Note that these geometric relations depend on three relative positions between $\mathbf{c}$ and $\boldsymbol{Q}_{BS}$: 1) $\mathbf{c}$ is on the open $\boldsymbol{Q}_{BS}$ so $\mathbf{q}^*$ coincides with $\mathbf{c}$, 2) $\mathbf{c}$ is off $\boldsymbol{Q}_{BS}$ and $\mathbf{q}^*$ is on the open $\boldsymbol{Q}_{BS}$, and 3) $\mathbf{c}$ is off $\boldsymbol{Q}_{BS}$ and $\mathbf{q}^*$ is at the end of $\boldsymbol{Q}_{BS}$. Among them, case 2) is the most general one. Our investigation starts with case 2) and we expect that case 1) and 3) can be integrated into the formula derived from case 2). The key observation is that for case 2), $\mathbf{cq}^* \perp \boldsymbol{L}(\mathbf{q}^*)$, where $\boldsymbol{L}(\mathbf{q}^*)$ is the tangent vector of $\boldsymbol{Q}_{BS}$ at $\mathbf{q}^*$ and $\perp$ is the perpendicularity. Thereby, the first two cases can be unified as

$$\mathbf{cq}^* \cdot \boldsymbol{L}\left(\mathbf{q}^*\right) = 0, \tag{2}$$

where $\cdot$ denotes the dot product, and for case 3), the boundary conditions can be added to the candidate parameter solution space to (2) to find the final ADMAM.

To sum up, the proposed ADMAM has the following four properties:

- it is in line with the subjective-based objective peak distortion assessment, so it is consistent and reliable,
- it totally avoids approximation, quantization or parameterization process on the original B-spline, therefore it is accurate and effective,
- when the B-spline-based operational rate-distortion optimal shape coding framework under the minimum-maximum criterion is exploited, it can guarantee the admissible distortion with the smallest bit-rate, and
- if the number of contour points is N_C, then it takes only $O(N_C)$ time for segment distortion calculation, which is the lowest complexity among the existing distortion measurements.

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